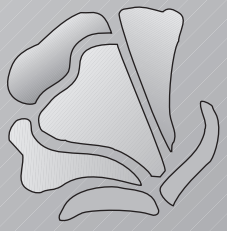
Teacher Guide
for
Mathematics



Golden State Examination

GSE

First-year Algebra

Geometry

High School Mathematics

•

2001

CALIFORNIA DEPARTMENT OF EDUCATION

This document has been prepared by the Sacramento County Office of Education and San Joaquin County Office of Education, under contract with the California Department of Education. For information about the Golden State Examination testing dates, registration materials and procedures, or about the Golden State Seal Merit Diploma, contact:

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Table of Contents

Introduction	2
Test Content for First-year Algebra, Geometry, and High School Mathematics	4
Test Structure for First-year Algebra, Geometry, and High School Mathematics	7
Scoring Guide for First-year Algebra, Geometry, and High School Mathematics	8
Sample Multiple-choice Questions for First-year Algebra	9
Sample Written-response Problem for First-year Algebra	14
Sample Student Work for First-year Algebra	15
Sample Multiple-choice Questions for Geometry	19
Sample Written-response Problem for Geometry	24
Sample Student Work for Geometry	25
Sample Multiple-choice Questions for High School Mathematics	29
Sample Written-response Problem for High School Mathematics	34
Sample Student Work for High School Mathematics	35

Introduction

Using the Golden State Examination Teacher Guide

The *Golden State Examination Teacher Guide* has been developed to provide essential information and preparation guidelines for teachers. The guide is intended to serve as an instructional aid in the classroom. It is divided into the following sections:

Test Content — outlines the content standards for which the Golden State Examination (GSE) questions have been developed.

Test Structure — describes the format of the test.

Scoring Guide — outlines the criteria used to score written-response problems.

Sample Questions — includes sample questions that represent the types of questions found on the exams. State content standards addressed by each sample question are identified.

Student Work — provides examples of student responses to written-response problems with teacher commentary.

Teachers are encouraged to reproduce portions or all of the guide for classroom use. Districts/schools also can use these materials with state standards for staff development.

Student Eligibility

The GSE in High School Mathematics is given during the winter test administration. The first-year algebra and geometry exams are given in the spring. Each examination may be taken only once.

Students who are enrolled in first-year algebra or geometry at the time the examinations are given and students who have taken the courses since the spring 2000 test administration may take the exams. Students completing a second-year integrated mathematics course may take either the first-year algebra exam, the geometry exam, or both. The high school mathematics examination is intended for students who have completed three years of high school mathematics. The exam covers the content included in the state standards for algebra I, geometry, algebra II, and probability and statistics. It is recommended that students taking the high school mathematics examination thoroughly review the test content for first-year algebra, geometry, and high school mathematics.

Test Preparation

All Golden State Examinations are aligned to state content standards. Teachers should review their curriculum and instructional activities for alignment to these standards.

Sound preparation for the Golden State Examinations should include classroom assignments that allow students to articulate the major ideas and concepts in the subject area being tested. Students also must be able to analyze information, apply knowledge, solve problems, and explain their solutions.

Reporting Results

All Golden State Examinations consist of two 45-minute sessions. Students who complete both sessions of the GSE in First-year Algebra, Geometry, or High School Mathematics receive an individual report of results. Scores for the multiple-choice and written-response portions of the exam are combined to produce the student's overall achievement level. There are six achievement levels. Students who achieve level six are awarded high honors; those who achieve level five are awarded honors; and those who achieve level four are awarded recognition. Students who achieve level three or below are acknowledged for their participation. Results for the winter administration are mailed to districts in May; results for the spring administration are mailed to districts in October.

Resource Document

The *Mathematics Content Standards for California Public Schools, Kindergarten Through Grade Twelve,* is available from the Publications Division, Sales Office, California Department of Education, P.O. Box 271, Sacramento, CA 95812-0271; 1-800-995-4099, ext. 6. It is also available at http://www.cde.ca.gov/board on the Internet.

Other Resources

The *Mathematics Framework for California Public Schools, Kindergarten Through Grade Twelve* (1999), is available from the Publications Division, Sales Office, California Department of Education, P.O. Box 271, Sacramento, CA 95812-0271; 1-800-995-4099, ext. 6.

Testing schedules and other information are available from the GSE coordinator in your district office, county office of education, or the California Department of Education at http://www.cde.ca.gov/statetests on the Internet.

Test Content for First-year Algebra, Geometry, and High School Mathematics

The content of the Golden State Examinations in First-year Algebra, Geometry, and High School Mathematics is based on the *Mathematics Content Standards for California Public Schools, Kindergarten Through Grade Twelve*. A complete listing of the mathematics standards is available at http://www.cde.ca.gov/board on the Internet.

Another useful source document is the *Mathematics Framework for California Public Schools, Kindergarten Through Grade Twelve*.

First-year Algebra

The GSE in First-year Algebra measures the use of basic algebraic skills and justifications in solving problems. Algebra I standards are the focus of the content. Other standards are identified when they serve as the foundation for the algebraic concept being assessed. Topics covered in the examination include but are not limited to:

Content	Algebra I Standards
Identify and use the arithmetic properties of integers, rational, irrational, and real numbers for evaluation and simplification of algebraic expressions, equations, and inequalities using:	1.0, 25.1, 25.2, 4.0, 25.3
• taking a root, exponents, opposites, reciprocals	2.0, 13.0
• factoring	12.0, 13.0
• monomials, polynomials	10.0, 12.0, 13.0
Solve equations, inequalities, and systems of equations using:	3.0, 4.0, 5.0, 9.0, 25.3
• linear and quadratic equations	3.0, 4.0, 16.0, 17.0
• factoring up to third degree polynomials	11.0, 22.0
• completing the square, quadratic formula	11.0, 14.0, 19.0, 20.0, 23.0
• absolute value, proportion	3.0, 5.0, 10.0, 13.0, 15.0
Graph linear and quadratic equations, inequalities, and functions involving:	6.0, 7.0, 16.0, 17.0
• parallel and perpendicular relationships, intercepts, intersection	6.0, 8.0, 21.0, 22.0
• distance and midpoint formulas, domain and range	7.0, 17.0
• alternative forms of linear equations, inequalities, making predictions	7.0, 18.0, 25.0
Apply the knowledge and skills of algebra, using appropriate problem-solving strategies	10.0, 13.0, 15.0, 23.0, 24.0, 25.0

Content	Grade 7 Standards: Measurement and Geometry
Relate geometric concepts involving:	
• similarity, perimeter, area, volume	2.0
• Pythagorean Theorem	3.0

Content	Probability and Statistics Standards	Statistics, Data Analysis, and Probability	
		Grade 7 Standards	Grade 6 Standards
Use probability and statistics for problems involving:			
• independent events, mean, median, mode	1.0, 6.0	1.0	1.0, 3.0
• scatterplots, histograms, stem and leaf plots	8.0	1.0	
• discrete data	3.0	1.0	1.0

Geometry

Computation, use of algebra, problem solving, proofs (formal and informal), and applications are integrated throughout the GSE in Geometry. Geometry standards are the focus of the content. Other standards are identified when they serve as the foundation for the geometric concept being assessed. Topics covered in the examination include but are not limited to:

Content	Geometry Standards	
Identify relationships among angles, lines, planes, and exterior angles related to:	1.0, 2.0, 3.0	
• triangle inequalities	6.0	
• parallel lines, intersecting lines, polygons	7.0	
• angles, lines, planes, exterior angles	13.0, 12.0, 1.0	
Identify and apply triangle and trigonometric relationships:	1.0, 12.0, 13.0, 2.0, 3.0	
• Pythagorean relationships, similarity, congruence	14.0, 15.0, 5.0, 6.0	
• special triangle relationships	20.0	
• area and perimeter of triangles	8.0, 10.0	**Trigonometry Standards**
• ratios: sine, cosine, tangent, law of sines	18.0, 19.0	12.0, 13.0, 19.0

Content	Geometry Standards	Algebra I Standards
Use properties of polygons other than triangles involving:	1.0, 2.0, 3.0	
• quadrilaterals	7.0, 10.0, 12.0	
• polygons with five or more sides	10.0, 12.0	
• relationships within polygons; sides, angles, midpoints, diagonals	13.0, 17.0	
• area, perimeter, similarity	11.0	
Identify and apply properties of circles involving:	1.0, 2.0, 3.0	
• angle and segment relationships	7.0, 12.0	
• basic relationships involving circles, area, circumference, sectors, arc measure, arc length	8.0, 11.0, 17.0	
Relate properties of coordinate and transformational geometry including:	22.0, 15.0	6.0, 7.0, 8.0, 9.0
• linear, nonlinear	15.0	
• translations, reflections, rotations, dilations	22.0	
Solve problems using the properties of three dimensional figures (including angles, surface area and volume of prisms, pyramids, cylinders, cones, and spheres)	8.0, 9.0, 11.0	

High School Mathematics

The GSE in High School Mathematics covers the major strands of algebra I, geometry, algebra II, and probability and statistics. Other standards are identified when they serve as the foundation for the algebraic and geometric concepts being assessed. Topics covered in the examination include but are not limited to:

- Algebra I: the skills and concepts summarized in the content description for the GSE in First-year Algebra. *(Algebra I 1.0 – 25.0)*
- Geometry: the skills and concepts summarized in the content description for the GSE in Geometry. *(Geometry 1.0 – 22.0)*
- Algebra II: expansion of the mathematical content and concepts of algebra I and geometry, including, but not limited to, working with algebraic solutions of problems in various content areas (e.g., systems of quadratic equations, logarithmic and exponential functions, the binomial theorem, and the complex number system). *(Algebra II 1.0 – 25.0)*
- Probability and statistics: the skills and concepts covered in an introduction to the study of probability, the processing of statistical information, sampling and statistical estimation, interpretation of data and graphs, and fundamental statistical problem solving. *(Probability and Statistics 1.0 – 8.0)*

Test Structure for First-year Algebra, Geometry, and High School Mathematics

The Golden State Examinations in First-year Algebra, Geometry, and High School Mathematics are two-part examinations, administered in two 45-minute sessions.

For the 2001 test administration, each session will consist of multiple-choice questions and a written-response question. This is a change from prior administrations.

The multiple-choice questions are designed to assess the student's breadth of knowledge. The questions emphasize concepts, principles, analysis, and the application of basic processes. The multiple-choice questions may require students to make connections among mathematical concepts or to organize information to arrive at the correct answer.

The multiple-choice portion of the examinations is machine-scored. Sample multiple-choice questions and answer keys are provided on pages 9–13 for first-year algebra, pages 19–23 for geometry, and pages 29–33 for high school mathematics. State content standards addressed by each question are identified for the purpose of this guide but do not appear on the examination.

The written-response problems require students to apply their mathematical skills and knowledge. Students are given problems and asked to provide a correct solution with all steps of their solution clearly shown. Students are also required to provide an explanation of how they arrived at the solution.

The written-response portion of the examinations is scored by experienced mathematics teachers and other professionals in the field. Sample written-response problems with student work and teacher commentary are presented on pages 14–18 for first-year algebra, pages 24–28 for geometry, and pages 34–38 for high school mathematics.

Teachers are encouraged to duplicate this guide for student use and to have students test themselves with the sample questions and problems.

Use of Calculators

For the 2001 test administration, calculators may be used during session two only. Calculators may not be used during session one. This is a change from prior administrations.

During session two, a scientific or graphing calculator may be used. Students are encouraged to bring their own calculators. Minicomputers, pocket organizers, or calculators with QWERTY (typewriter) keyboards are not allowed. Calculators cannot be shared.

Scoring Guide for First-year Algebra, Geometry, and High School Mathematics

The written-response portion of the Golden State Examinations in First-year Algebra, Geometry, and High School Mathematics is scored using criteria based on the general scoring guide below.

A detailed training package for scorers, addressing what students are expected to accomplish, is developed to score each student response.

Score Point 4

The student response thoroughly accomplishes the task. The response:

- shows thorough understanding and use of the central mathematical idea(s)
- includes appropriate and accurate mathematical computations
- presents mathematical knowledge and ideas clearly and skillfully, using combinations of mathematical symbols and/or visual means as supporting evidence

Score Point 3

The student response substantially accomplishes the task. The response:

- shows an essential grasp of the central mathematical idea(s)
- includes appropriate and generally correct mathematical computations
- presents mathematical knowledge and ideas clearly with supporting evidence

Score Point 2

The student response partially accomplishes the task. The response:

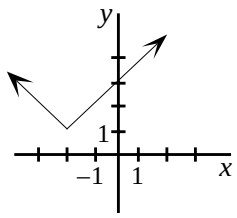
- shows a limited grasp of the central mathematical idea(s)
- may include incomplete and/or misdirected mathematical computations
- presents mathematical knowledge and ideas in an unclear manner or without supporting evidence

Score Point 1

The student response makes little or no progress toward accomplishing the task. The response:

- shows little or no grasp of the central mathematical idea(s)
- includes mathematical computations that are incorrect or inappropriate
- presents mathematical knowledge and ideas in a barely (if at all) comprehensible manner

Sample Multiple-choice Questions for First-year Algebra

1. 

Note: Figure not drawn to scale.

Which of the following equations represents the graph above?

A. $y = -|x + 2| + 1$

B. $y = |x + 2| + 1$

C. $y = |x - 2| - 1$

D. $y = |x + 2| - 1$

Algebra I Standards – 3.0, 6.0

2. The sides of a red cube are numbered 1, 2, 3, 4, 5, and 6. The sides of a blue cube are numbered 2, 4, 6, 8, 10, and 12. When the cubes are tossed, what is the probability that the sum of the two numbers is less than 7?

A. $\frac{6}{36}$ or $\frac{1}{6}$ B. $\frac{9}{36}$ or $\frac{1}{4}$

C. $\frac{6}{12}$ or $\frac{1}{2}$ D. $\frac{8}{12}$ or $\frac{2}{3}$

Probability and Statistics Standard – 1.0

3. What is the *y*-intercept of a line that passes through (–5, 6) and has an *x*-intercept of 3?

A. $\frac{4}{9}$ B. 2

C. $2\frac{1}{4}$ D. 3

Algebra I Standard – 7.0

4. Of the following three polynomials, which are not factorable?

I. $x^2 + 2x + 18$

II. $x^2 + 4x$

III. $x^2 + 10x - 96$

A. I only B. III only

C. I and III only D. II and III only

Algebra I Standards – 11.0, 19.0, 20.0

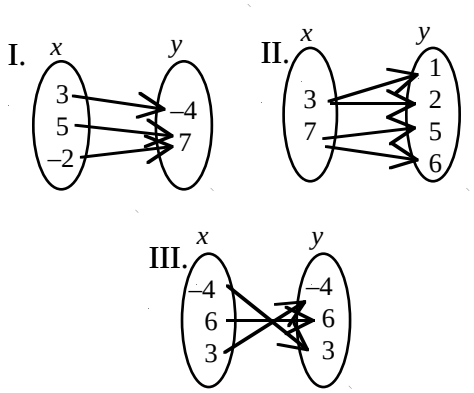
5. Solve $|3x + 7| = 11$.

A. – 6 only B. $\frac{4}{3}$ only

C. $\frac{4}{3}$ and $-\frac{4}{3}$ D. $\frac{4}{3}$ and – 6

Algebra I Standard – 3.0

6. Using the following mapping, which represent functions?



A. I only

B. II only

C. I and III only

D. II and III only

Algebra I Standards – 16.0, 18.0

7. Graciela wanted to buy a new winter coat. The store was offering a 30% discount plus an extra $6 discount for paying cash. Graciela bought a coat for cash and paid $80.80. Which of the following equations could be used to find the **original price** of the coat, x?

A. $0.70x + 6 = 80.80$

B. $0.70x - 6 = 80.80$

C. $x - 0.30(x + 6) = 80.80$

D. $x - 0.30(x - 6) = 80.80$

Algebra I Standard – 5.0

8.

x	–1	2	3	7
y	33	21	17	1

From the chart above, what is the value of y when $x = 12$?

A. 68 B. 19

C. –19 D. –48

Algebra I Standard – 17.0

9. How many different values of k exist so $x^2 + kx - 15$ is factorable using integers?

A. 1 B. 2

C. 4 D. 6

Algebra I Standards – 11.0, 19.0, 20.0

10. Pipe A can fill a tank in 8 hours. When both pipe A and pipe B are used, the tank is filled in 5 hours. How long, in hours, would it take pipe B alone to fill the tank?

A. $6\frac{1}{2}$ B. 9

C. 13 D. $13\frac{1}{3}$

Algebra I Standards – 15.0, 9.0, 10.0

11. Jack and Jill sat at the ends of an 18-foot long seesaw where the fulcrum was placed in the middle of the board. By placing a 20-pound weight 3 feet in front of Jill, they were in balance. If Jack weighs 140 pounds, which equation could be used to determine Jill's weight, x, in pounds?

    A. $140(9) = 9x + 6(20)$

    B. $140(9) = 9x + 3(20)$

    C. $140(9) = 9x + (x - 3)20$

    D. $140(18) = 18x + 15(20)$

    Algebra I Standard – 5.0

12. When $\frac{p+2}{4p} - \frac{3p-1}{6p^2}$ is simplified, the numerator is:

    A. $3p^2 - 2$ B. $4p + 1$

    C. $3p^2 + 2$ D. $-2p + 3$

    Algebra I Standards – 12.0, 13.0

13. What is the distance between $(6, -1)$ and $(-3, 7)$?

    A. 6.71 B. 8.54

    C. 10.82 D. 12.04

    Grade 7: Algebra and Functions Standard – 3.3
    Grade 7: Measurement and Geometry Standard – 3.0
    Algebra I Standard – 2.0
    Geometry Standard – 17.0

14. Solve for t:

    $2(t + 3) - 3(-2t + 5) = \frac{1}{2}(16t - 18)$

    A. $\{0\}$

    B. $\{9\}$

    C. {real numbers}

    D. no real solution

    Algebra I Standard – 4.0

15. What is the value of c if $x^2 + 7x + c$ is a perfect square?

    A. $\sqrt{7} \approx 2.65$ B. $\frac{7}{2}$

    C. $\frac{49}{4}$ D. 49

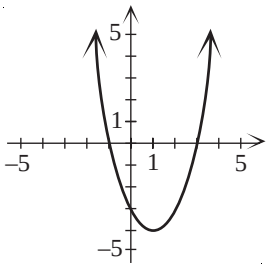
    Algebra I Standards – 11.0, 14.0

16. Roxanne invested $18,200, part at 6.5% per year and the remainder at 7.8% per year. If she earned $487.50 from her 6.5% investment, how much did she earn from her 7.8% investment?

    A. $406.25 B. $487.50

    C. $585.00 D. $834.60

    Algebra I Standards – 15.0, 9.0, 10.0

17. 

The figure shows the graph of $y = x^2 - 2x - 3$. The expression $x^2 - 2x - 3 < 0$ when

A. $x < 0$

B. $x < 1$ or $x > 3$

C. $x > -4$

D. $-1 < x < 3$

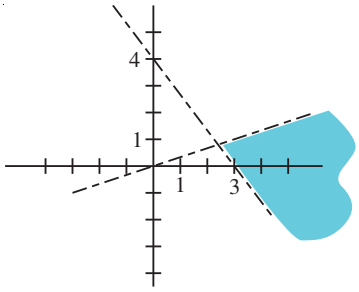
Algebra I Standard – 21.0

18. The vertex of the parabola represented by $y = x^2 + 8x + 13$ is in which quadrant?

A. I B. II

C. III D. IV

Algebra I Standards – 21.0, 22.0

19. 

Note: Figure not drawn to scale.

The shaded region is determined by which pair of inequalities?

A. $4x + 3y < 12$
$x - 3y < 0$

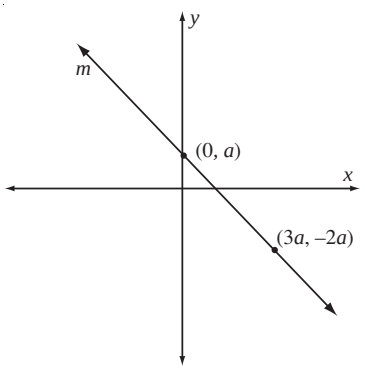
B. $4x + 3y < 12$
$x - 3y > 0$

C. $4x + 3y > 12$
$x - 3y < 0$

D. $4x + 3y > 12$
$x - 3y > 0$

Algebra I Standards – 7.0, 6.0, 9.0

20.



Note: Figure not drawn to scale.

Which of the following is an equation for line *m*?

A. $y = -ax + a$

B. $y = -x + a$

C. $y = \frac{-3}{2}x + a$

D. $y = \frac{-3}{2}ax + a$

Algebra I Standards – 7.0, 6.0

First-Year Algebra Answer Key

1.	B	4.	A	7.	B	10.	D	13.	D	16.	D	19.	D
2.	A	5.	D	8.	C	11.	A	14.	C	17.	D	20.	B
3.	C	6.	C	9.	C	12.	C	15.	C	18.	C		

Sample Written-response Problem for First-year Algebra

Problem

Estaban is heating water for a science experiment. The starting temperature of the water is 52°C. After 6 minutes of heating, the water's temperature reaches 61°C. The temperature continues to rise at a constant rate. Using two different methods, determine the total number of minutes it would take the water to reach boiling point (100°C).

Algebra I Standards – 5.0, 6.0, 7.0

What Students Are Expected to Accomplish Mathematically

Students are asked to find the length of time it would take water to reach boiling point if, after 6 minutes of heating, the water went from 52°C to 61°C and continued to rise at a constant rate. To thoroughly accomplish the task, students must clearly communicate the sequence of steps taken to reach an accurate conclusion using two different approaches. Responses should demonstrate knowledge of using ratios and proportions, graphing, equations, charts, and/or tables.

Sample Student Work for First-year Algebra

Score Point 4

STUDENT RESPONSE*

Equation

a) Using two different methods determine the number of minutes it will take the water to reach boiling point (100°)

b) Let x = minutes to take for boiling point

c) 52 + x(1.5) = 100

52 + 1.5x = 100

52 - 52 + 1.5x = 48

1.5x ÷ 1.5 = 48 ÷ 1.5

x = 32

d) It takes 32 minutes to reach boiling

e) 52 + 32(1.5) = 100 ?

52 + 48 = 100 ?

100 = 100 ✔

chart

Time	Temperature
start	52°
start + 6min	61°
start + 12min	70°
start + 18min	79°
start + 24min	88°
start + 30min	97°
start + 32min	100°
start + 36min	106°

every 2 minutes the temperature goes up 3°C.

Explanation - If every 2 minutes the temperature goes up 3°C (9°C in 6 minutes), it takes about 32 minutes from the start (52°C). Every minute the temperature goes up 1.5°C, and the starting point is 52°C, the equation should be 52 + x(1.5). We want to find out how long it will take before 100°C, so the total equation is 52 + x(1.5) = 100. When you do the equation, x = 32 minutes. I also did a chart which tells how much time it took to reach 100°C (it goes up by 6 minutes). The answer would be: it will take 32 minutes for the water to reach boiling point (100°C) if the water started at 52°C.

COMMENTARY

The response demonstrates a thorough accomplishment of the task. An equation is derived and solved completely and accurately. A second method using a chart with a full and accurate explanation is given. Both methods provide specific evidence to illustrate the solutions. A paragraph explaining both methods is complete and clearly written.

* The student response has been typed as written, with the student's own content, grammar, spelling, and punctuation.

Sample Student Work for First-year Algebra

Score Point 4

STUDENT RESPONSE*

starting temperature = 52°C temperature after 6 minutes = 61°C.

① $61 - 52 = 9$ — temperature risen after 6 minutes

$\frac{9}{6} = \frac{3}{2}$ — ratio of temperature risen to minutes heated is 3/2.

$100 - 52 = 48$ — # of degrees water needs to rise

$\frac{48}{x} = \frac{3}{2}$ — Equation used to find # of minutes

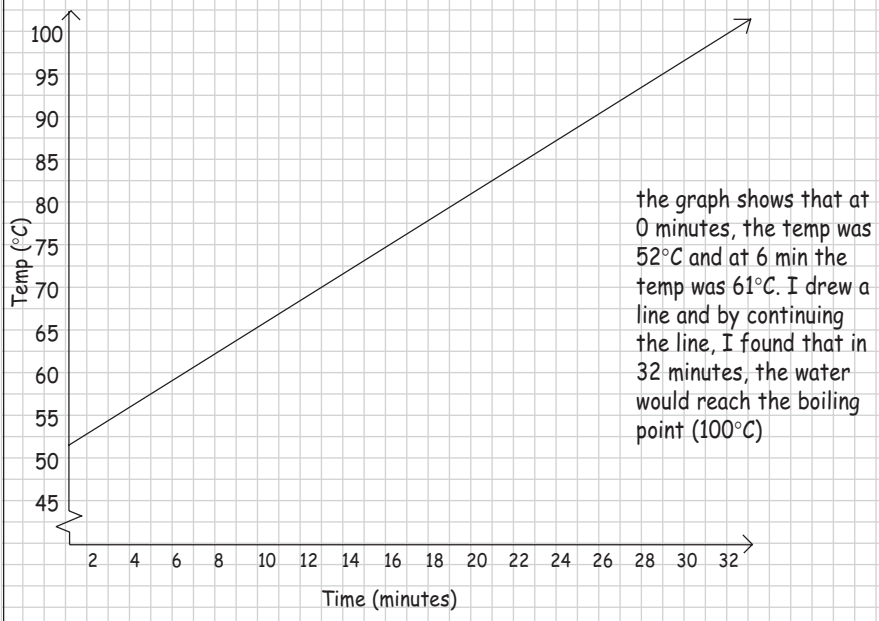
$3x = 48(2)$ — Cross multiply

$3x = 96$ — Simplify

$\frac{3x}{3} = \frac{96}{3}$ — Divide by 3 to isolate x

$x = 32$ — # of minutes to reach boiling point

Answer: It takes a total of 32 minutes for the water to reach the boiling point (100°C).



the graph shows that at 0 minutes, the temp was 52°C and at 6 min the temp was 61°C. I drew a line and by continuing the line, I found that in 32 minutes, the water would reach the boiling point (100°C)

Answer: It takes a total of 32 minutes for the water to reach the boiling point (100°C)

COMMENTARY

The response demonstrates thorough accomplishment of the task. The work is complete, accurate, and clearly explained. The first method establishes a proportion that is solved and fully explained. A second method using a graph is clearly labeled and described. Each method includes specific evidence to support the solutions.

* The student response has been typed as written, with the student's own content, grammar, spelling, and punctuation.

Sample Student Work for First-year Algebra

Score Point 3

STUDENT RESPONSE*

method ①

61 – 52 = 9 degrees

9 degrees in 6 minutes

100 – 61 = 39 degrees

make a ratio of

$$\frac{\text{change in degrees}}{\text{minutes}}$$

$$\frac{9}{6} = \frac{39}{x}$$ cross multiply

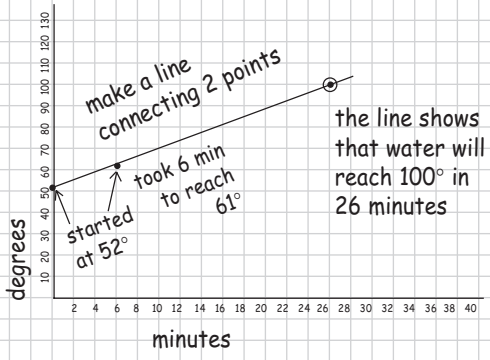
$$\frac{234}{9} = \frac{9x}{9}$$

x = 26 minutes

26 minutes for water to gain boiling point.

method ②

make a graph to find how long it would take for the water to reach 100°C



water will reach boiling point in 26 minutes

COMMENTARY

The response demonstrates substantial accomplishment of the task by using two methods, ratio and proportion, followed by a graphing solution. The response contains a common error in calculating from 61°, which results in 26 minutes to reach 100°C. The response is appropriate and generally correct, but the first 6 minutes are omitted in arriving at the solution. The graph shows the line from 52°, but it is a little off, causing the indication of an incorrect answer of 26 minutes.

* The student response has been typed as written, with the student's own content, grammar, spelling, and punctuation.

Sample Student Work for First-year Algebra

Score Point 3

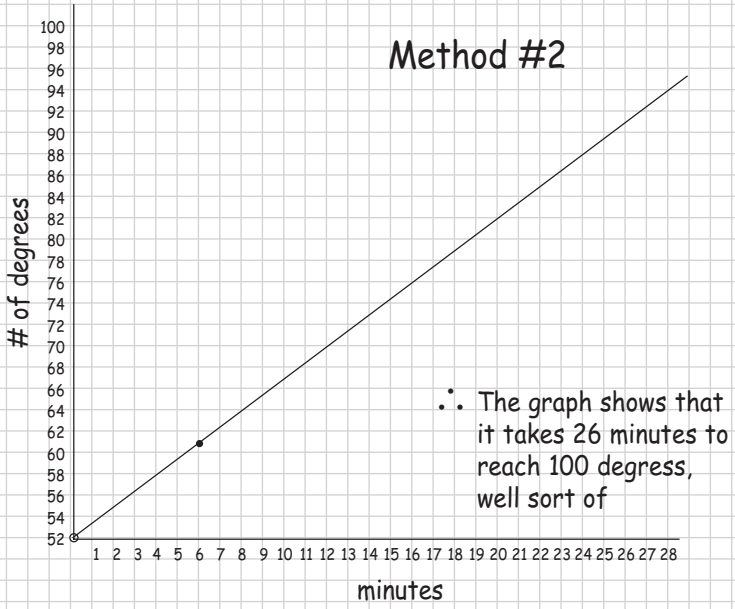
STUDENT RESPONSE*

$\frac{\text{After 6 min}}{\text{water raised 9}} = \frac{\text{X = how much time}}{\text{Needs to rais 39° more}}$ Method #1

than I cross multiply

x = 26 min.

∴ I takes 26 more minutes to reach 100°C



COMMENTARY

The response demonstrates substantial accomplishment of the task by using two methods, ratio and proportion, followed by a graphing solution. The response includes a common error in calculating from 61°, which results in 26 minutes to reach 100°C. The response is appropriate and generally correct in stating that "it takes 26 more minutes to reach 100°C." The graph would show the correct answer of 32 minutes if it had been clearly labeled.

* The student response has been typed as written, with the student's own content, grammar, spelling, and punctuation.

Sample Multiple-choice Questions for Geometry

1. Two planes intersect at line $\overleftrightarrow{AB}$. Each plane contains a line that does not intersect $\overleftrightarrow{AB}$. Which of the following terms describes the relationship between these two lines?

   A. skew

   B. coplanar

   C. perpendicular

   D. intersecting

   Geometry Standards – 1.0, 7.0

2. ΔMNO is similar to ΔXYZ. If $MN = 15$, $MO = 20$, $NO = 30$, $XY = 4k - 2$, and $XZ = 4k + 4$, find XY.

   A. 6 B. 12

   C. 15 D. 18

   Geometry Standards – 4.0, 12.0

3. Line AE intersects line CD at point B, $m\angle ABC = (x-2)(2x+2)$, and $m\angle DBE = (x-1)(x+4)$. What is the measure of $\angle CBE$?

   A. 5 B. 36

   C. 144 D. 176

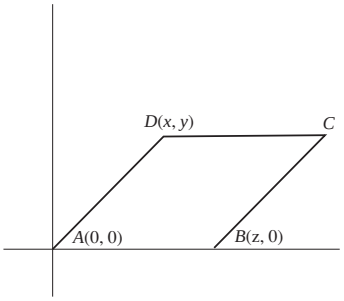
   Geometry Standards – 12.0, 13.0
   Algebra I Standards – 1.0, 4.0, 10.0

4. In parallelogram $ABCD$, the measure of $\angle A$ is $(2x + 5)°$ and the measure of $\angle C$ is $(4x - 59)°$. The measure of $\angle B$ is

   A. 32° B. 69°

   C. 83° D. 111°

   Geometry Standards – 7.0, 12.0, 13.0

5. 

   Note: Figure not drawn to scale.

   Figure $ABCD$ is a rhombus, with vertices located at the coordinates shown. What is the slope of a line perpendicular to side $\overline{BC}$?

   A. $\frac{x}{y}$ B. $\frac{y}{x}$

   C. $-\frac{x}{y}$ D. $-\frac{y}{x}$

   Geometry Standards – 7.0, 17.0

6. Which of the following points are vertices of a square with one side tangent to the circle $(x - 3)^2 + y^2 = 9$?

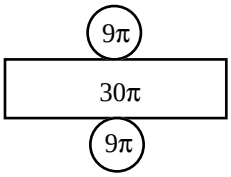
   A. (0, 0) (6, 0)
   (3, 3) (3, –3)

   B. (–4, –1) (0, 1)
   (–4, 1) (0, –1)

   C. (6, 2) (10, 2)
   (6, –2) (10, –2)

   D. (6, 6) (9, 3)
   (6, 0) (3, 3)

Geometry Standard – 17.0

7. 

Note: Figure not drawn to scale.

What is the volume of the cylinder formed by the pattern with the given areas?

A. 141.37 B. 565.48

C. 848.23 D. 2664.79

Geometry Standard – 9.0

8. The diameter of the base of a cone is 18 and the height is 14. What is the volume of the cone?

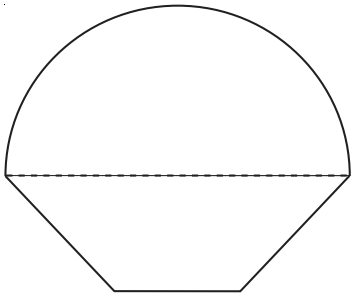
   A. $378\pi \approx 11187.5$

   B. $588\pi \approx 1847.3$

   C. $972\pi \approx 3053.6$

   D. $1134\pi \approx 3562.6$

Geometry Standard – 9.0

9. 

Note: Figure not drawn to scale.

The floor of a theater is made up of a semicircle with radius 50 feet and one-half of a regular hexagon as shown. How much carpet should be purchased to cover this floor? All answers are in square feet.

A. $1250\pi + 625\sqrt{3}$

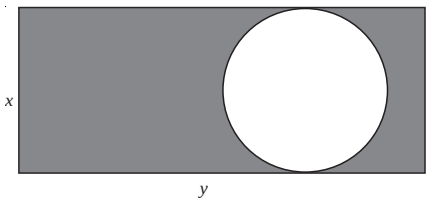
B. $1250\pi + 1875\sqrt{3}$

C. $2500\pi + 1875\sqrt{3}$

D. $2500\pi + 3750\sqrt{3}$

Geometry Standards – 8.0, 12.0

10.



Note: Figure not drawn to scale.

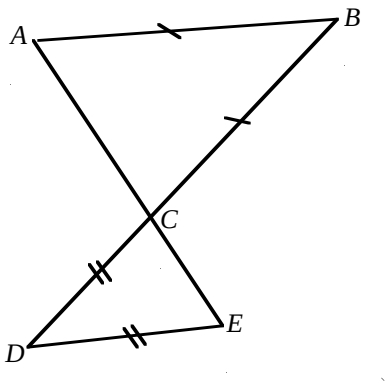
A round ice-skating rink is built inside a rectangular arena as shown in the figure. The management needs to order new material to cover the floor represented by the shaded area. Which of the following algebraic expressions represents this shaded area?

A. $xy - \pi x^2$ B. $\pi x^2 - xy$

C. $xy - \frac{\pi x^2}{4}$ D. $xy - \frac{\pi x^2}{2}$

Geometry Standards – 21.0, 8.0, 10.0

11.



Note: Figure not drawn to scale.

What is the measure of $\angle B$ if $m\angle A = 4x$, $m\angle D = 6x - 2$, $\overline{AB} \cong \overline{BC}$, and $\overline{DC} \cong \overline{DE}$?

A. 52 B. 67

C. 72.8 D. 76

Geometry Standard – 12.0

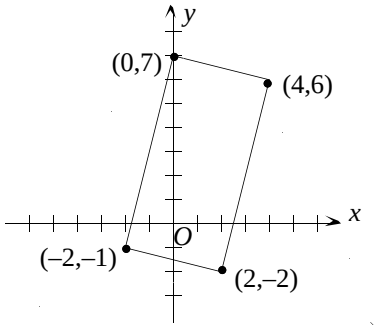
12. The endpoints of one diagonal of a square are (–5, 2) and (7, –6). What is the equation of the line containing the other diagonal?

A. $3x + 2y = -1$ B. $3x + 2y = 2$

C. $3x - 2y = -1$ D. $3x - 2y = 7$

Geometry Standard – 17.0

13. The area of the rectangle shown below is



A. $6\sqrt{5} \approx 13.42$

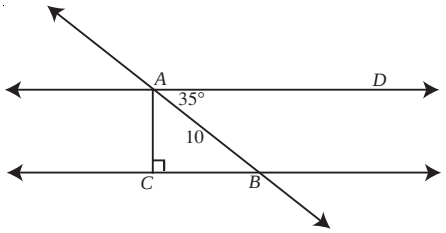
B. $2\sqrt{85} \approx 18.44$

C. 32

D. 34

Geometry Standard – 17.0

14.



Note: Figure not drawn to scale.

$\overleftrightarrow{AD} \parallel \overleftrightarrow{CB}$ and cut by transversal $\overline{AB}$.
$\overline{AC} \perp \overleftrightarrow{CB}$. AB = 10, $\angle DAB = 35°$.
Which of the following expressions could be used to determine the length of $\overline{AC}$?

A. 10 cos 55°

B. 10 sin 55°

C. 10 cos 35°

D. 10 tan 35°

Geometry Standards – 18.0, 7.0

15. What is the area, in square inches, of the circle that is circumscribed about an equilateral triangle that has a perimeter of 36 inches?

A. $18\pi \approx 56.55$

B. $27\pi \approx 84.82$

C. $36\pi \approx 113.10$

D. $48\pi \approx 150.80$

Geometry Standards – 20.0, 8.0

16. The apothem (drawn from the center, perpendicular to a side) of a regular hexagon has a length of 6. What is the perimeter of the hexagon?

A. $2\sqrt{3} \approx 3.46$

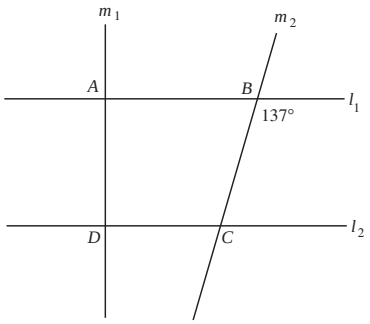
B. $4\sqrt{3} \approx 6.93$

C. $12\sqrt{3} \approx 20.78$

D. $24\sqrt{3} \approx 41.57$

Geometry Standards – 20.0, 12.0, 15.0

17.



Note: Figure not drawn to scale.

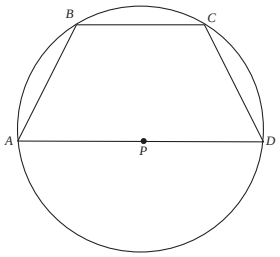
In the figure, $l_1 \parallel l_2$, m_1 and m_2 are transversals and $m_1 \perp l_2$. If AD = 15.8 cm, what is BC in cm?

A. 10.8 B. 16.9

C. 21.6 D. 23.2

Geometry Standards – 7.0, 19.0

18.



Note: Figure not drawn to scale.

Trapezoid *ABCD* is inscribed in circle *P*. If $\overline{AB} \cong \overline{BC} \cong \overline{CD}$ and $AP = 12$, what is the measure of the median of the trapezoid?

A. 6 B. 12

C. 18 D. 24

Geometry Standard – 21.0

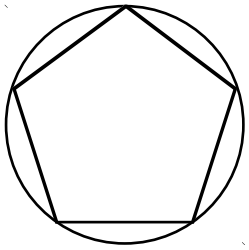
19. First rotate the triangle bounded by the vertices (0, 0), (0, 5), (4, 0) about the origin 180°. Then, reflect the resulting image across the *x*-axis. What quadrant contains the final image of the triangle after both transformations?

A. quadrant I B. quadrant II

C. quadrant III D. quadrant IV

Geometry Standard – 22.0

20.



Note: Figure not drawn to scale.

A circle of area 25π circumscribes a regular pentagon as shown. To the nearest tenth, what is the length of an arc joining two adjacent vertices?

A. 4.3 B. 5.0

C. 5.9 D. 6.3

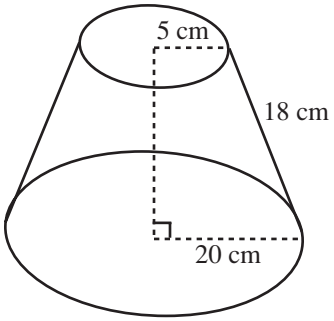
Geometry Standards – 8.0, 21.0

Geometry Answer Key

1.	B	4.	D	7.	A	10.	C	13.	D	16.	D	19.	B
2.	D	5.	C	8.	A	11.	D	14.	A	17.	D	20.	D
3.	C	6.	C	9.	B	12.	D	15.	D	18.	C		

Sample Written-response Problem for Geometry

Problem



Note: Figure not drawn to scale.

The top portion of a right circular cone has been cut off by a plane parallel to the base, resulting in the figure. Find the volume of the remaining portion.

$\left(\text{Volume of a cone } = \frac{1}{3}Bh \right)$

Geometry Standards – 20.0, 15.0, 8.0

What Students Are Expected to Accomplish Mathematically

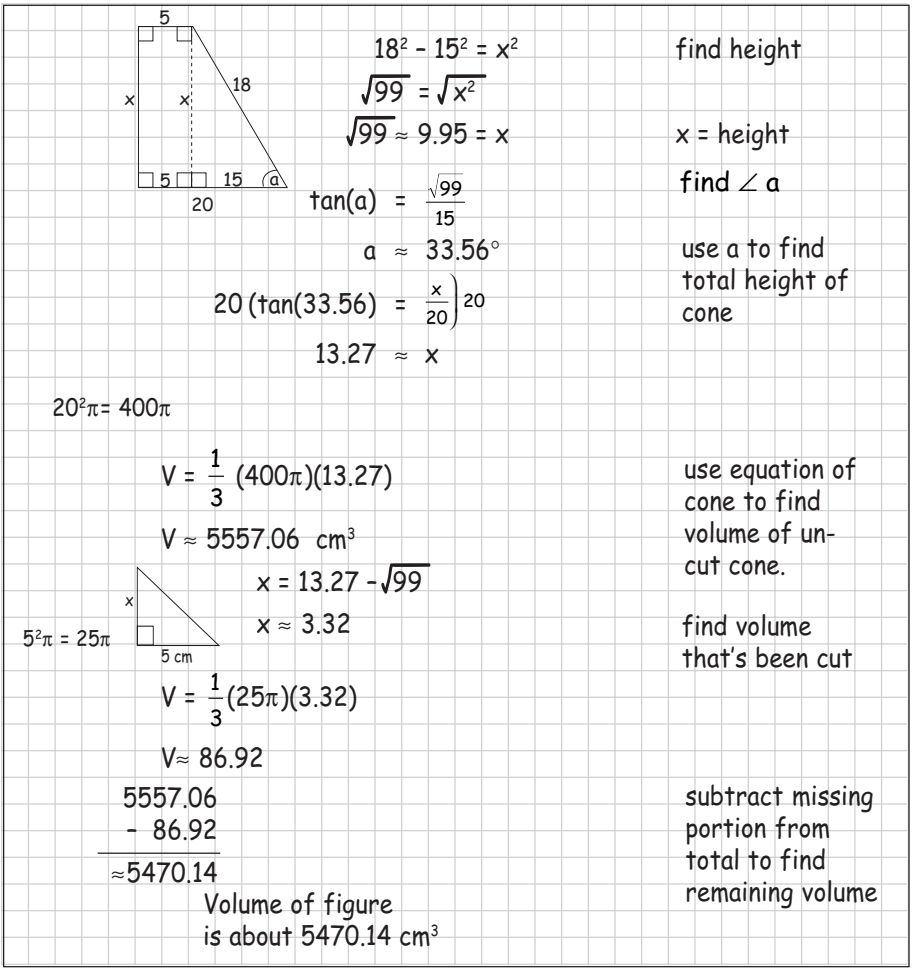
Students are asked to find the volume of the bottom part of a right circular cone (frustum) when the top has been cut off by a plane that is parallel to the base.

To thoroughly accomplish the task, students must clearly communicate the sequence of steps taken to reach a conclusion that the volume of the figure is approximately 5470.23 cm^3 or 1743.69π cm^3. The response should demonstrate knowledge of the height and volume of cones, the area of the circle, and the process of subtracting the volume of the top cone from the whole cone to arrive at the volume of the remaining figure. Some students used trigonometry while others used the Pythagorean theorem to determine the height of the cone.

Sample Student Work for Geometry

Score Point 4

STUDENT RESPONSE*



COMMENTARY

This response demonstrates thorough and accurate accomplishment of the task. The angle formed at the base of the cone is used with the tangent function to determine the height of the cone with a base radius of 20 cm. The volume of the cone with a 20 cm radius minus the volume of the cone with a 5 cm radius is used to derive the volume for the remaining figure. All calculations are accurate and are supported by a step-by-step explanation.

* The student response has been typed as written, with the student's own content, grammar, spelling, and punctuation.

Sample Student Work for Geometry

Score Point 4

STUDENT RESPONSE*

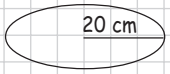
Find the volume

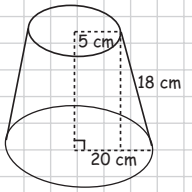
$V = \frac{1}{3}Bh$

$B = \pi r^2$

$= \pi (20)^2$

$= 400\pi \text{ cm}^2$





$115^2 + h^2 = 18^2$

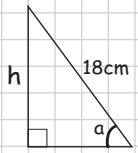
$225 + h^2 = 324$

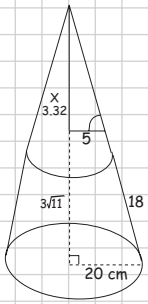
$h^2 = 99$

$h = \sqrt{99}$

$h = 3\sqrt{11} \text{ cm}$

$\approx 9.95\text{cm}$





$\tan 33.56° = \frac{x}{5}$

$3.32 = x$

$\cos a = \frac{15}{18}$

$a \approx 33.56°$

top portion $V = \frac{1}{3}(25\pi)(3.32)$

$\approx 25.64\pi$

whole $V = \frac{1}{3}(400\pi)(13.27)$

$= 1769.33\pi$

remaining $V \approx 1769.33\pi - 25.64\pi = \boxed{1743.69\pi \text{ cm}^3}$

First, I worked for the base of a whole cone and looked for the height of the remaining portion. I then looked for ∠a, because I needed it to find out x. I solved for the volume of whole cone next and subtracted the volume of top portion to get the volume of the remaining portion.

COMMENTARY

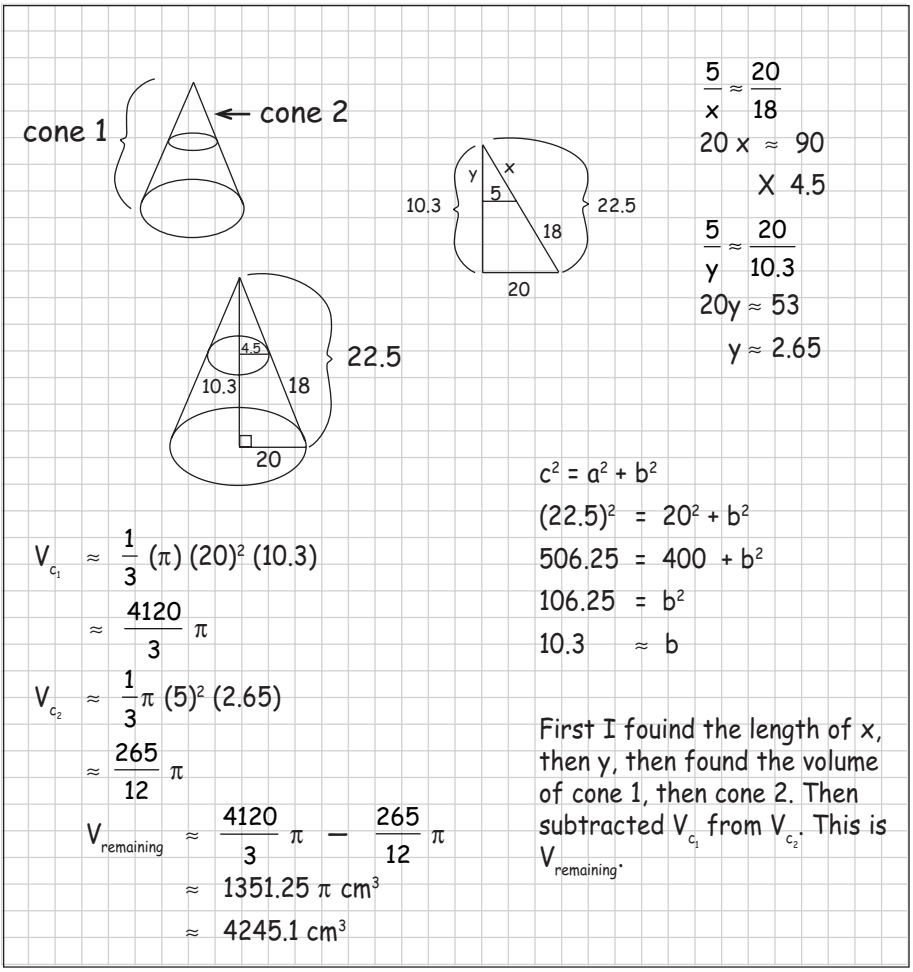
This response demonstrates thorough and accurate accomplishment of the task. The angle formed at the base of the cone is used with the tangent function to determine the cone's height. The solution is determined in terms of π. The volume of the cone with a radius of 5 cm on the base is subtracted from the cone with a 20 cm radius base to obtain the volume of the given figure. All calculations are accurate and clearly presented. Specific evidence supports the solution.

* The student response has been typed as written, with the student's own content, grammar, spelling, and punctuation.

Sample Student Work for Geometry

Score Point 3

STUDENT RESPONSE*



COMMENTARY

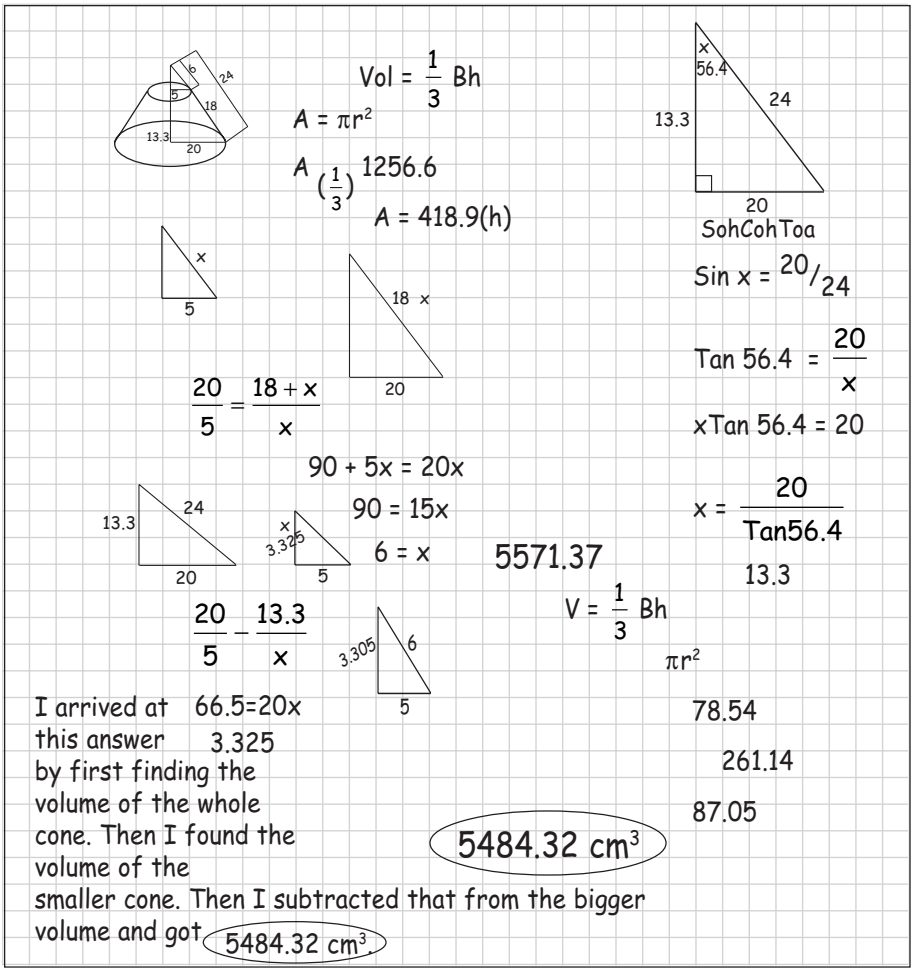
This response demonstrates substantial accomplishment of the task. The concept of proportion is used to determine the slant height of the smaller cone; however, one of the proportions used is incorrect and causes the solution to be miscalculated. All calculations and support are clearly presented.

* The student response has been typed as written, with the student's own content, grammar, spelling, and punctuation.

Sample Student Work for Geometry

Score Point 3

STUDENT RESPONSE*



COMMENTARY

This response demonstrates substantial accomplishment of the task. Trigonometry was used to determine the height of the cone. Work shows a clear understanding of finding the volume by subtracting the volume of the smaller cone from the volume of the big cone to determine the volume. Communication is not always clear, and there are rounding errors in the calculations.

* The student response has been typed as written, with the student's own content, grammar, spelling, and punctuation.

Sample Multiple-choice Questions for High School Mathematics

1. As treasurer for your junior class, you sold x tickets to the dance in advance and y tickets at the door. If tickets are \$5 paid in advance and \$6 at the door, the total amount you collected can be represented as

   A. $30xy$ B. $11(x + y)$

   C. $30(x + y)$ D. $5x + 6y$

   Grade 7: Algebra and Functions Standard – 1.1

2.

Weight (ounces)	Price
<1	\$0.32
1.1–2	\$0.55
2.1–3	\$0.78
3.1–4	\$1.01

   If the cost of mailing a letter first-class was as shown in the table above, then letters weighing 1.1 ounces, 1.5 ounces, 1.9 ounces, or 2 ounces would all cost \$0.55. If the pattern shown in the table continues up to 11 ounces, what would be the cost of mailing a 9.7 ounce letter?

   A. \$2.16 B. \$2.32

   C. \$2.39 D. \$2.62

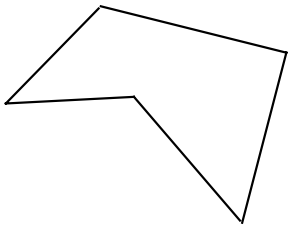
   Algebra I Standards – 17.0, 5.0

3. A math teacher recently calculated some statistics on a geometry test she had given to her students. The mean score for the class was 78 and the standard deviation was 9. Assuming the data is normally distributed, what percent of the students received scores of 87 or above?

   A. 9% B. 13%

   C. 16% D. 22%

   Probability and Statistics Standard – 5.0

4. 

   Note: Figure not drawn to scale.

   The figure shown above has an area of six square units. A similar figure is constructed so that each segment of the boundary is three times as long as the corresponding side in the figure shown. The area of the new figure, in square units, is

   A. 18 B. 36

   C. 54 D. 108

   Geometry Standards – 4.0, 11.0

5. The function $f(x)$ is an increasing function and has a single asymptote at $y = 5$. Which of the following could be an equation for $f(x)$?

   A. $f(x) = 2x + 6$

   B. $f(x) = \left(\frac{1}{2}\right)^x + 5$

   C. $f(x) = 2^x + 5$

   D. $f(x) = 2^x + 6$

   Algebra II Standard – 12.0

6. Which of the following equations of a parabola has a vertex at (–4, 16) and passes through the origin?

   A. $y = (x - 4)^2 + 16$

   B. $y = (x + 4)^2 + 16$

   C. $y = -(x - 4)^2 + 16$

   D. $y = -(x + 4)^2 + 16$

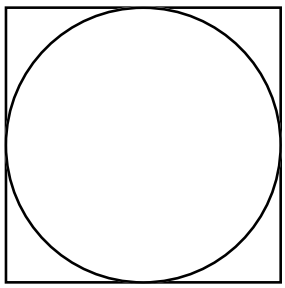
   Algebra II Standard – 9.0

7. Kevin's grandmother placed $1,000 in a savings account on the day he was born in 1982. It has been drawing a 9% interest rate compounded annually. How much should Kevin have on his birthday in the year 2000 if the interest remained constant every year?

   A. $4,327.63 B. $4,717.12

   C. $19,620.00 D. $34,200.00

   Grade 7: Number Sense Standard – 1.7

8. 

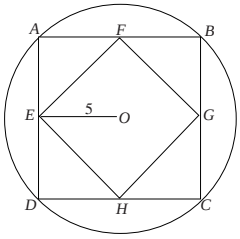
   If the area of the given square is 25, what is the area of the inscribed circle?

   A. 5π B. $\frac{25\pi}{4}$

   C. 10π D. 25π

   Geometry Standards – 8.0, 21.0

9.



Note: Figure not drawn to scale.

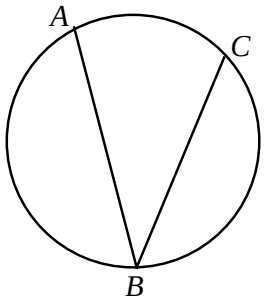
Square *ABCD* is inscribed in a circle with center *O*. Square *EFGH* has a diagonal length of 10. What is the circumference of the circle?

A. 31.4 B. 44.4

C. 78.5 D. 157.0

Geometry Standards – 8.0, 21.0

10.



Note: Figure not drawn to scale.

The figure shown above is a circle with a radius of six units, and the measure of $\angle ABC$ is 30°. The length of arc *AC* is approximately

A. 3 units B. 6 units

C. 19 units D. 108 units

Geometry Standard – 21.0

11. A bag contains four one-dollar bills, two five-dollar bills, and two ten-dollar bills. Two randomly selected bills are removed from the bag. What is the probability that neither bill is a ten-dollar bill?

A. $\frac{2}{7}$ B. $\frac{15}{28}$

C. $\frac{2}{3}$ D. $\frac{3}{4}$

Probability and Statistics Standards – 1.0, 2.0

12. A computer company's change in the number of employees for three consecutive years is, respectively, 20% increase, 30% increase, and 20% decrease. What is the total percent change (rounded to the nearest percent)?

A. 25% B. 30%

C. 70% D. 87%

Grade 7: Number Sense Standard – 1.7

13. Suppose the total daily cost C, of producing x tables, is given by $C = 3.5x + 200$. If the company produced a certain number of tables at a cost of $42,200, how many tables were produced?

A. 147,900 B. 147,700

C. 12,114 D. 12,000

Algebra I Standard – 5.0

14. The function $F(x)$ satisfies the following conditions:
- has a y-intercept at $y = 5$
- has an x-intercept at $x = –3$
- has exactly one asymptote at $y = 10$

Which of the following conclusions could be made about $F(x)$?

A. $F(x)$ is a constantly decreasing function.

B. $F(x)$ cannot be a linear function.

C. $F(x)$ has a slope of $\frac{5}{3}$.

D. $F(x)$ could be a parabola which opens downward.

Algebra II Standard – 24.0

15. The line l passes through the points $(3, –1)$ and $(1, \frac{1}{3})$. Another point on the line l is

A. $(–1, 1)$ B. $(\frac{3}{5}, \frac{3}{5})$

C. $(1, \frac{–7}{3})$ D. $(2, \frac{2}{3})$

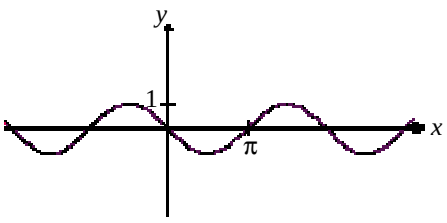
Algebra I Standard – 7.0

16. Given ΔABC with $A(0, 0)$, $B(5, 5\sqrt{2})$, and $C(–4, 3)$, what is the measure of $\angle CAB$ to the nearest degree?

A. 60 B. 88

C. 90 D. 92

Geometry Standards – 12.0, 17.0
Trigonometry Standard – 13.0

17. 

What is the value of this periodic function when $x = \frac{19\pi}{2}$?

A. –1 B. 0

C. $\frac{1}{2}$ D. 1

Algebra I Standard – 16.0
Algebra II Standard – 24.0

18. In the first-period algebra class, the average test score was 73.4, and in the second-period class, the average score was 69.4. There were 31 students in the first-period class and 37 in the second-period class. What was the average score for the two classes combined?

A. 71.2 B. 71.3

C. 71.4 D. 71.5

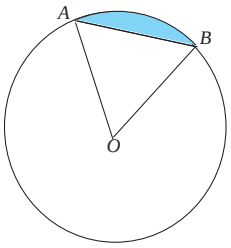
Probability and Statistics Standard – 6.0

19. Leo is planning a field trip for sixth and seventh graders. No more than 30 sixth and seventh graders can go. The total number of seventh graders must be less than three times the number of sixth graders and greater than two times the number of sixth graders. What is the greatest number of seventh graders that can go on the field trip?

A. 20 B. 21

C. 22 D. 23

Algebra II Standard – 2.0

20. 

Note: Figure not drawn to scale.

ΔAOB is equilateral, and the radius of the circle is 12 cm. What is the area, in square centimeters, of the shaded region?

A. $4\pi - 18\sqrt{3}$

B. $24\pi - 18\sqrt{3}$

C. $4\pi - 36\sqrt{3}$

D. $24\pi - 36\sqrt{3}$

Geometry Standards – 8.0, 21.0

High School Mathematics Answer Key

1.	D	4.	C	7.	B	10.	B	13.	D	16.	B	19.	C
2.	C	5.	C	8.	B	11.	B	14.	B	17.	B	20.	D
3.	C	6.	D	9.	B	12.	A	15.	B	18.	A		

Sample Written-response Problem for High School Mathematics

Problem

Gina is making fudge to send to her family. She is going to make her own personalized boxes by taking a rectangular piece of cardboard covered with foil that measures 12 inches by 6 inches. She will cut congruent squares out of the corners, folding up all four sides of remaining cardboard and taping where the edges touch. She wants to be as generous as possible. She is hoping to make boxes capable of holding the most fudge. To the nearest 1/8 inch, what are the dimensions of the cutout squares?

Geometry Standard – 8.0; Algebra II Standard – 10.0

What Students Are Expected to Accomplish Mathematically

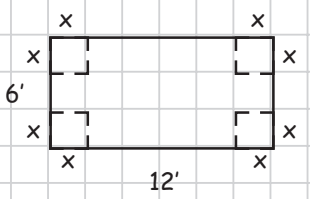
To fully accomplish the task, students must show thorough understanding of maximizing the volume of a rectangular prism. The formula for the volume must be developed and the maximum determined within the constraints of the problem. Students should include mathematical support or justification for the maximum value.

Note: Two of the four student responses presented in this guide used the first derivative to solve the problem. It is an appropriate method for the prompt; however, it is not necessary or expected in order to successfully accomplish this task.

Sample Student Work for High School Mathematics

Score Point 4

STUDENT RESPONSE*

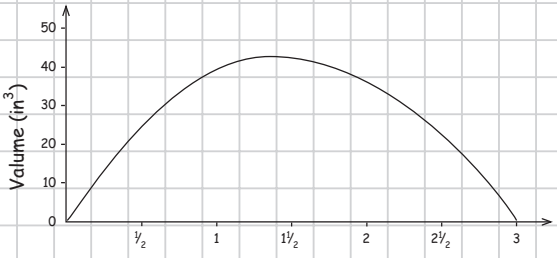


Let x be the unkown side of the cutout square

Volume = Length x Width x Height

= (12 – 2x) • (6 – 2x) • x

Domain for x : 0 < x < 3, since neither length, width, nor height is equal to 0



It seems like the side length of the cutout squares should be ranged from 1' to 1½' in order to mazimize the volume. And we'll go find out the answer by testing 1', 1 ⅛', 1¼', and 1⅜', 1½'

X = 1'

Volume = [12' – 2(1')] • [6' – 2(1')] • 1' = 40in.^{3}

x = 1 ⅛'

Volume = [12' – 2(1 ⅛')'] • [6' – 2(1⅛')] • 1⅛' ≈ 41.1328 in.^{3}

x = 1¼'

Volume = [12' – 2(1¼')] • [6' – 2(1¼')] • 1¼' = 41.5625 in.^{3}

x = 1⅜'

Volume = [12' – 2(1⅜')] • [6' – 2(1⅜')] • 1⅜' ≈ 41.3359 in.^{3}

x = 1½'

Volume = [12' - 2(1½')] • [6' - 2(1½')] • 1½' = 40.5 in.^{3}

∴ In order to have the max. volume, the dimension for the cutout squares should be 1¼' by 1¼'

COMMENTARY

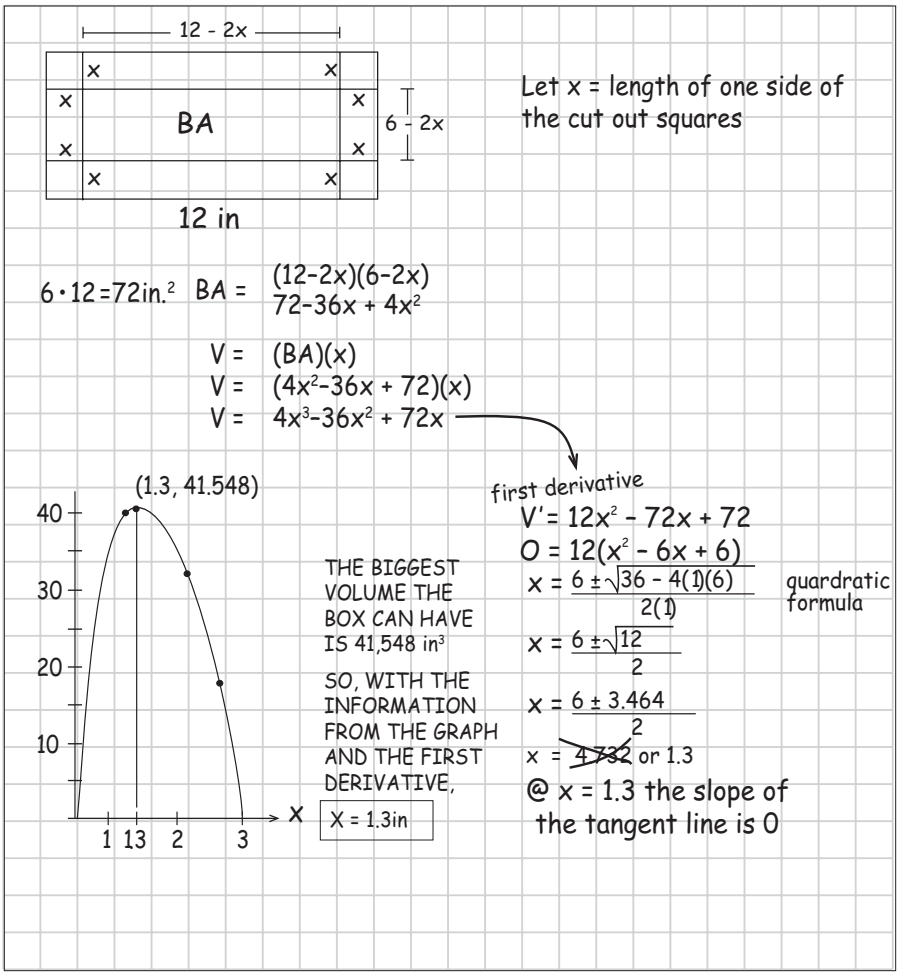
This response shows a thorough understanding of maximizing the volume of a rectangular prism. The correct volume function is stated and its domain restricted to reflect the constraints of the given conditions. The maximum value is justified graphically and by using a table set at increments of 1/8 inch. Work is clearly shown and communicated.

* The student response has been typed as written, with the student's own content, grammar, spelling, and punctuation.

Sample Student Work for High School Mathematics

Score Point 4

STUDENT RESPONSE*



COMMENTARY

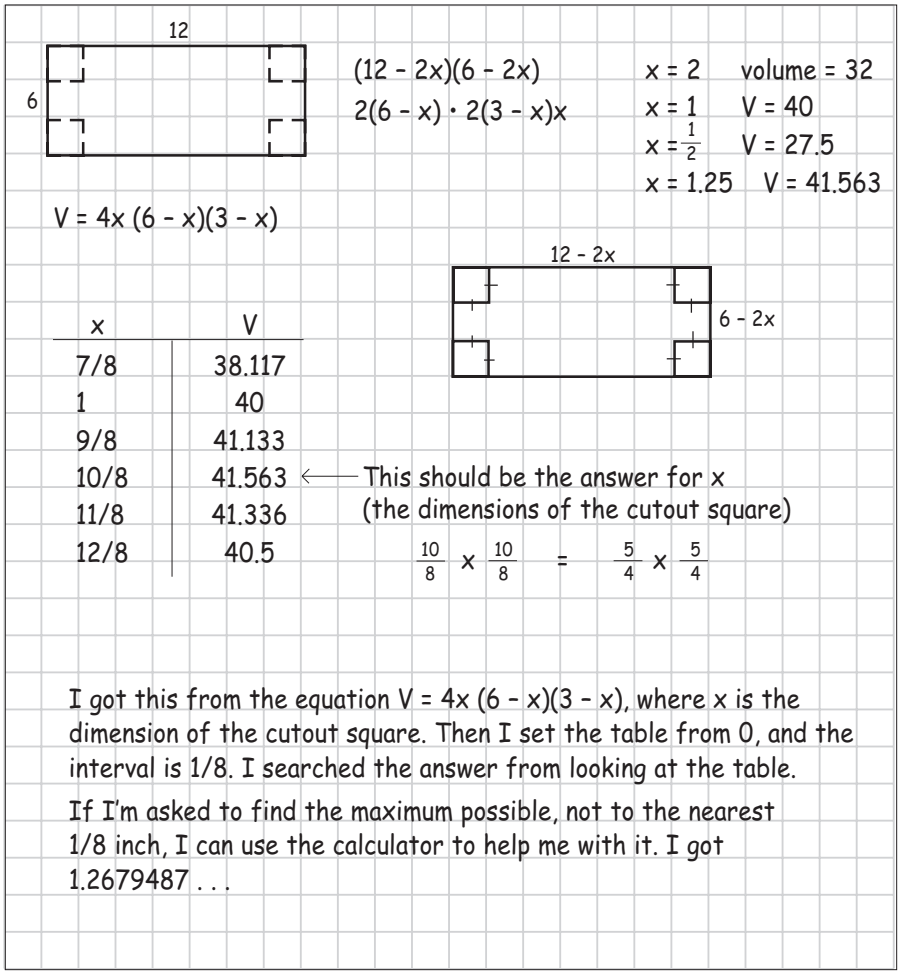
This response shows a thorough understanding of maximizing the volume of a rectangular prism. The correct volume function is stated. The domain restriction is addressed by an accurate graph. The maximum value is justified using a combination of a graph and the first derivative of the volume function.

* The student response has been typed as written, with the student's own content, grammar, spelling, and punctuation.

Sample Student Work for High School Mathematics

Score Point 3

STUDENT RESPONSE*



COMMENTARY

The response shows a substantial understanding of maximizing the volume of a rectangular prism. The correct volume function is stated and analyzed in table form in 1/8 inch increments. The response does not clearly address the domain restrictions.

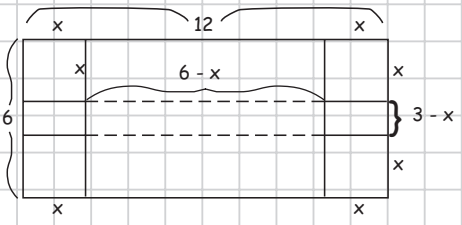
* The student response has been typed as written, with the student's own content, grammar, spelling, and punctuation.

Sample Student Work for High School Mathematics

Score Point 3

STUDENT RESPONSE*

Let x in. be the length of 1 side of the square



The volume of the rectangular box is:

$(12 - 2x)(6 - 2x)\,x$

$= 4x(6 - x)(3 - x)$

$= 4x(18 - 6x - 3x + x^2)$

$= 4x(18 - 9x + x^2)$

$= 72x - 36x^2 + 4x^3$

The maximum volume of the box = maxima of the differential of the volume

$\frac{d}{dx}(72x - 36x^2 + 4x^3) = 0$

$72 - 72x + 12x^2 = 0$

$6 - 6x + x^2 = 0$

$x = \frac{6 \pm \sqrt{36 - 24}}{2} = \frac{6 \pm 2\sqrt{3}}{2} = 3 \pm \sqrt{3}$

$= 4.732$ in. or 1.268 in. (meaningless)

$\therefore$ The value of $x = 1.268$ in. or $1\frac{1}{4}$ in.

COMMENTARY

This response shows a substantial understanding of maximizing the volume of a rectangular prism. The correct volume function is stated and the critical values of the first derivative of the volume function are correctly calculated. The response does not justify why the chosen value is a maximum.

* The student response has been typed as written, with the student's own content, grammar, spelling, and punctuation.